

## Shock Pulse Mitigation in Various Organic Foams

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### Synopsis

The mechanical behavior of three kinds of organic foams, each at two different densities, was experimentally investigated under conditions of pulsed one-dimensional strain shock loading. The input pulse width in each experiment was nominally 0.1  $\mu$ sec, and the input stress level (as referenced to quartz) was varied between 10 and 23 kbar. The materials studied were polyurethane foam at bulk densities of 0.33 and 0.21 g/cc, syntactic foam (phenolic microballoons dispersed in a resin binder) at 0.66 and 0.23 g/cc, and polystyrene bead foam at 0.091 and 0.049 g/cc. Specimen thicknesses varied between 1.0 and 6.5 mm. It was found that the pulse duration was greatly lengthened and that the peak stress was decreased (accounting for both impedance mismatch and attenuation effects) by factors of between about 8 and 500, depending upon the type of foam, its thickness, and its density.

### INTRODUCTION

It is recognized that the mechanical properties of a material can be significantly altered by its distention or by the introduction of voids. The outstanding characteristic of such distended materials, usually called foams or porous solids, is their low density relative to the bulk parent material. Foamed materials can be made of polymers, metals, or ceramics.

As a consequence of the unique properties of foams, they have been used in such diverse practical applications as flotation devices, thermal insulators, and, more recently, in certain structural applications that take advantage of their high strength-to-weight and rigidity-to-weight ratios. We are concerned generally with the property of foams as mitigating materials to absorb impact energy and with their ability to serve as cushions, or in situations requiring shock isolation.

Measurement and interpretation of the response of shock-loaded foam materials are relatively recent, with the first comprehensive articles on this subject appearing in the literature about a decade ago.<sup>1</sup> Most of the work during this period has been confined to the study of metal foams and directed to the determination of such information as the relevant constitutive equations, Hugoniot data, and equations of state. Very little dynamic work has been done on organic foams.<sup>2</sup> The usual approach to

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these problems has been primarily experimental in nature with an empirical representation of data. The empirical approach is generally used since many of the mechanisms of plastic deformation (crush-up) are not fully understood, although significant progress has been made.<sup>3</sup> In addition, the equations defining the behavior are too complex for usual design purposes since they involve time-dependent deformation, e.g., viscoelastic or elastic-viscoplastic,<sup>2</sup> of a complicated cell structure.

The specific purpose of this work is to examine experimentally the effectiveness of certain selected organic foams to produce relatively low-amplitude, long-time output pressure pulses from high-amplitude very short-duration, one-dimensional strain input pulses, e.g., the kind delivered by high explosives or thin flying plates. An associated goal is to determine to what extent existing experimental techniques are satisfactory for such investigations. The present effort is restricted to studies conducted under ambient temperature conditions. This work is considered somewhat preliminary and is not intended to be an exhaustive investigation, and one must use some care in extrapolating the results to other experimental geometries and input pulse parameters.

Three primary mechanisms are responsible for the mitigation of the stress amplitude and are qualitatively described below. The first decrease in stress arises from the impedance mismatch, i.e., reduction, from the driver plate to the foam specimen. This effect occurs essentially immediately upon impact, and the final equilibrium stress state is a point on the Hugoniot curve. (As used here, the term "Hugoniot" is applied to foams to mean the locus of final macroscopic (continuum) pressure-relative volume or pressure-particle velocity states of shocked material deduced from experimental observations and from the use of the Rankine-Hugoniot conservation equations. This Hugoniot represents actual rather than average shocked states of the material only if the scale of porosity is such that equilibrium is attained within the time scale of the experiment.)

The second mechanism of stress pulse mitigation can be explained on the basis of shock decay, or shock attenuation, arising from energy dissipation. A foamed material contains voids or pores, which are collapsed and crushed during the transmission of strong pressures through it. The nonelastic crushing process in highly compressible materials involves large relative volume changes and causes significant internal energy dissipation in the form of heat (sometimes called "waste heat"). Therefore, in a shock pulse of finite duration, the internal mechanical energy, and hence the shock pressure, are continuously degraded as the shock progresses.

The third mechanism of stress pulse mitigation arises from the loading-unloading process in the foam specimen. Since crushing of the voids takes time, shock waves moving through a foamed specimen are slowed, allowing rarefactions (unloading acoustic waves) from the impacted surface which travel comparatively rapidly through the compacted material, to overtake and relieve the shock front during the shock transit time interval in the specimen. This relatively rapid erosion of the shock front has the effect of

redistributing the rate of delivery of the entering impact momentum, which must be conserved, over the entire thickness of the specimen. Such a condition thus prevents a rapid momentum transfer leaving the specimen and consequently reduces the induced peak stress. In contrast, a stress pulse introduced into many solid metal specimens would tend to retain its initial shape with relatively little dispersion because the input shock and following rarefactions move at nearly equal velocities.

### EXPERIMENTAL

The experimental work was conducted with the use of a modified 90-mm gun.<sup>4</sup> The significant advantage of using a gun facility in this study was the ability to control precisely the input pulse shape, duration, and amplitude by the variation of the velocity, thickness, and composition of the flyer plate assembly on the head of the projectile. The apparatus and test arrangement enabled an 89-mm-diam. flat-faced projectile to be launched under conditions of intermediate vacuum ( $\sim 10^{-2}$  torr) to eliminate air cushion effects, and at varying velocities up to 0.32 mm/ $\mu$ sec (in these tests). The geometry of a flyer plate-gun experiment as used in this investigation is schematically shown in Figure 1.

To provide planar impacts with negligible background stress, all experiments used rigid, open-cell ( $>98\%$ ) carbon foam of density 0.1 g/cc as flyer plate backup, and no adhesive was used between the plate and the foam. The flyer plate was 2024-T4 aluminum, and its thickness was nominally 0.25 mm; the thickness of the carbon foam plate backup was nominally 20 mm. More details on dimensions are given below, and a more complete description of the design, construction, and conduct of thin flyer plate-gun experiments in general is presented elsewhere.<sup>5</sup>

Projectile velocity, tilt angle, time of impact, and point of first impact were calculated using results from piezoelectric pins, as shown in Figure 1. The stress-time profiles were monitored using quartz crystal pressure

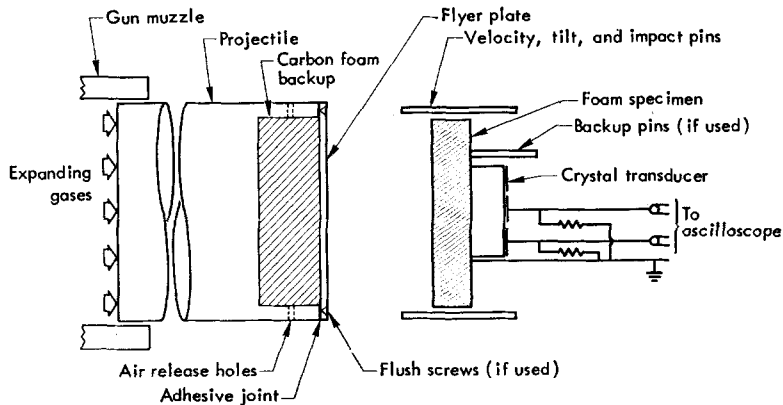


Fig. 1. Experimental geometry (schematic).

TABLE I. Specimen Description and Characterization

Type and designation	Remarks
Polyurethane foam Stafoam BC1220	Rigid urethane, <sup>a</sup> cell size = 0.120 mm ± 0.020 mm (range); material made from a polyether tetrol derived from pentaerythritol, propylene oxide, and a toluene diisocyanate-polyester prepolymer, using tertiary amine catalysts.
Polyurethane foam Rigifoam 7004-6C	Rigid urethane, <sup>a</sup> cell size = 0.225 mm ± 0.030 mm (range). R component: Atlas Atpol 2406, trimethylol propane, and glycerol. T component: prepolymer of caprolactone polyester and toluene diisocyanate [hydroxyl number 605 (polyester); amine equivalent of T component = 145]. Tertiary amine catalysts.
Syntactic foam	Filler: 23 phr phenolic microballons (Union Carbide designation BJO-0930); chemical composition (pbw)—C, 66.92% H, 5.12% N, 0.38% O, 27.58%; density—bulk = 0.105 g/cc; liquid displacement = 0.25 g/cc; particle size range = 0.005–0.127 mm; average = 0.043 mm. Resin system: thermosetting epoxy, consisting of base resin (Shell designation Epon 826); 10 phr diethylene triamine; 1 phr foaming suppressant (Union Carbide designation SAG-47). Cure: room temperature with deaeration.
Syntactic-coated foam	Filler: 80 phr phenolic microballons (Union Carbide designation BJO-0930); same data as for microballons above. Resin system: thermosetting epoxy, acrylate type consisting of 40% pbw ethylene glycol dimethacrylate and 60% pbw epoxy-terminated acrylate (similar to Shell designation Epocryl U-12); 0.25% pbw cobalt naphthanate 6%, 1.0% pbw <i>t</i> -butyl hydroperoxide-70, and microballons are added to resin; the mixture is vacuum molded to achieve uniform low density. Cure: 16 hr at elevated temperatures in argon atmosphere. Coating: poly(vinyl butyral), 0.25 mm thick per side; its function is to provide a sealed surface for future bonding of foam.
Polystyrene bead; foam; 10-56	Manufactured from polystyrene beads (Dow Chemical designation SD-505; formerly Pelaspan 222L), using hot gas (N <sub>2</sub> ) as the transfer medium in the heated mold. Chemical composition of SD-505 (pbw)—C, 92.7% H, 7.2% plus minor components. Cell size: varies from 0.08 to 0.3 mm with cell wall thickness about 0.005 mm; the variation in bead size is about ±100% from nominal while the cell size within a bead is quite uniform but varies from bead to bead depending upon the degree of nucleation; within the limits of this variation no cell elongation is visible. Blowing agent: a mixture of approximately 75% <i>n</i> -pentane and 25% isomers. Volatiles: using ASTM Method D-2362, total volatiles = 1.6%.
Polystyrene bead foam; SN/100	Manufactured in the same manner as for the above PSB foam; cell wall thickness is about 0.003 mm; 1.9% volatiles; all other description as above.

<sup>a</sup> The foaming of the rigid polyurethane is accomplished by the release of CO<sub>2</sub> generated from the reaction of excess diisocyanate with water.

TABLE II  
Summarized Data

Experiment no.	Flyer plate driver system		Foam specimen			Impact parameters		Results	
	Thickness, mm	Approx. pulse width, $\mu$ sec	Type and designation	Bulk density, g/cc	Thickness, mm	Plate velocity ( $\pm L_{0.5}$ ), mm/ $\mu$ sec	Tilt angle at impact, min	Shock velocity from experiment, mm/ $\mu$ sec	Wave velocity from ultrasonics, mm/ $\mu$ sec
1	0.29	0.09	polyurethane; Stafoam BC1220	0.33	2.81	0.153 (0.001)	8.77	1.56	1.35
2	0.28	0.09	polyurethane; Rigi foam 7004-6C	0.21	1.40	0.156	— <sup>a</sup>	0.58	—
2A	0.28	0.09	same	0.21	2.43	0.271 (0.003)	2.51	1.05	1.14
2B	0.25	0.08	same	0.21	5.23	0.270 (0.002)	7.39	1.87	—
3	0.28	0.09	syntactic	0.69	1.03	~0.15 <sup>a</sup>	— <sup>a</sup>	1.15	—
3A	0.28	0.09	same	0.66	2.00	0.155	— <sup>a</sup>	1.66	—
3B	0.25	0.08	same	0.66	3.23	0.317 (0.004)	7.38	1.62	1.93
4	0.25	0.08	syntactic-coated	0.23	3.24	0.316 (0.003)	5.77	0.40	—
4A	0.29	0.09	same	0.23	3.26	0.131 (0.005)	6.26	0.88	0.81
4B	0.31	0.10	same	0.23	6.47	0.221 (0.009)	5.11	0.94	—
5	0.28	0.09	polystyrene bead; R-10-56	0.091	2.97	0.304 (0.010)	1.27	—	1.09
6	0.25	0.08	polystyrene bead; SN/100	0.049	2.81	0.310 (0.003)	13.67 <sup>b</sup>	—	1.02

<sup>a</sup> The rasterized oscilloscopes and/or time interval counters did not function properly.

<sup>b</sup> One of the four tilt pins did not report, and another one was suspect.

transducers.<sup>6-8</sup> The experimental ranges of amplitude and duration associated with the stress pulses were outside the recently observed<sup>9,10</sup> anomalous response region of these crystal pressure transducers. The determination of shock velocity in the specimen was also determined using the quartz transducers and the impact time pins.

### MATERIAL DESCRIPTION AND PREPARATION

The materials investigated were polyurethane foam at bulk densities of 0.33 and 0.21 g/cc, syntactic foam (phenolic microballoons dispersed in a resin binder) at 0.66 and 0.23 g/cc, and polystyrene bead foam at 0.091 and 0.049 g/cc. All the foams are essentially isotropic. The details of sample characterization are given in Table I. Specimen thicknesses were varied between 1.0 and 6.5 mm and are given in Table II. The bulk densities were determined separately on each of the specimens actually tested and are also given in Table II.

The specimens were prepared by first machining 76-mm-diam. disks from the several foams to the nominal thicknesses desired. Each sample was then gently hand-lapped on a flat granite block surfaced with No. 600 carborundum paper. All observable foreign particles were carefully removed from the specimens by application of an air jet. By this procedure, flatnesses to <0.01 mm T.I.R. and parallelisms to <0.05 mm T.I.R. were obtained.

### RESULTS AND DISCUSSION

The data from these experiments are summarized in Table II. The description of the flyer plate driver system and the specimens used, as given in the table, are for the most part self-explanatory.

Usually in such experiments, the flyer plate impacts a given sample with tilt angles less than  $1\frac{1}{2}$  min. As noted in Table II, the tilts are generally significantly greater than this limit. The reason for the relatively poor tilts achieved was due to the difficulty in properly prealigning the target with the projectile because of the physical nature of the foam specimens.

As stated previously, the shock transit times in our experiments were obtained from the start times of the quartz transducer oscilloscope traces. However, because of the very low stress (and hence signal) levels developed, the external trigger mode of the oscilloscopes occasionally did not function properly, and they operated in the internal trigger mode only. Thus, in two experiments, determination of the sample shock velocity could not be made. The values given in the last column in Table II are longitudinal acoustic wave velocities determined at 400 kHz using a differential ultrasonic technique with a poly(methyl methacrylate) (PMMA) buffer.<sup>11</sup> The specimens actually tested were used for these ultrasonic tests.

The stress-time records as calculated from the quartz transducer traces are shown in Figures 2 through 5. All data are normalized to a flyer plate velocity of 0.3 mm/ $\mu$ sec to facilitate comparison. Normalization was ac-

completed by assuming that the stress is directly proportional to the projectile velocity. This procedure is thus only correct for materials whose Hugoniot are linear within the plate velocity (and hence stress) range under consideration. The validity of such a simple approach was verified in experiments that examined various methods of flyer plate backup construction using carbon foam.<sup>5</sup> The extension of the validity to this foam study is a reasonable assumption considering the range of stresses involved.

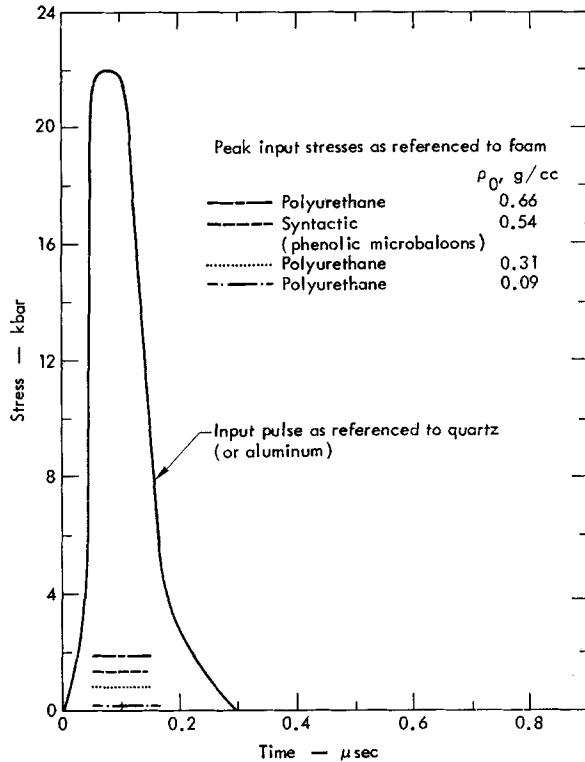


Fig. 2. Stresses at impact. The approximate Hugoniot data for the foams are obtained from ref. 12. Data are normalized to a plate velocity of 0.3 mm/ $\mu$ sec.

Figure 2 plots the stresses obtained at time of impact between the aluminum flyer plate and the foam specimen. The input pulse was obtained from associated experimental work,<sup>5</sup> and the pulse stress is referenced to quartz. Since the shock impedances between quartz and aluminum are essentially the same, the peak stress (22 kbar) is also that in the aluminum flyer plate. The approximate peak stress decreases because of the flyer plate-specimen impedance mismatch are also indicated on the figure for two foam types and various densities.<sup>12</sup> It can be seen that significant stress changes occur from these mismatches.

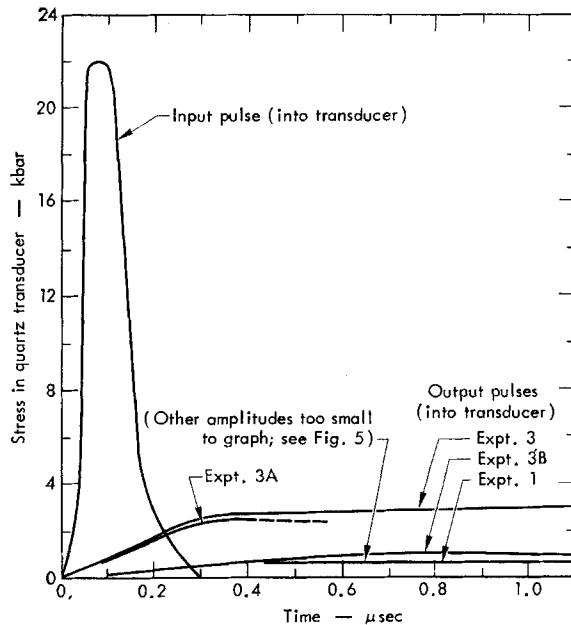


Fig. 3. Input and output stresses. Data are normalized to a plate velocity of 0.3 mm/ $\mu$ sec. The dashed line indicates a questionable portion of the record.

Figure 3 plots to the same scales the results of both input and some of the output stress-time records. Figures 4 and 5 give the output stress-time profiles for the foams investigated. Note the stresses are given in reference to quartz. Although specific data are not available, the output stresses as referenced to the several foams can be approximately obtained by dividing the various ordinates by 2 (because the shock impedance of the quartz is considerably greater than those of the foams).

It can be seen that the "background" stress generated by the flyer plate backup carbon foam can be neglected in these tests. Specifically, the results from experiment 4B on Figure 5 show that such background stress is at least no greater than about 0.04 kbar.

Good confirmation in pressure results was obtained in the repeated experiment using coated syntactic foam material (experiments 4 and 4A). Since impact velocities were different in the two tests, the normalization process referred to above appears satisfactory. Moreover, the two experiments were performed approximately six months apart, using samples that had been prepared at the same time. Thus, it appears that effects on the stress-time results from the leakage of gases entrapped in the foam cells are negligible for these high-pressure short-duration tests.

It should be noted that for this study, involving short pulses, no forerunner precursor waves<sup>13,14</sup> were observed in the output pulses.

There is a reasonable consistency in the data, and one can make some ranking in the stress mitigation results with respect to type of foam, its



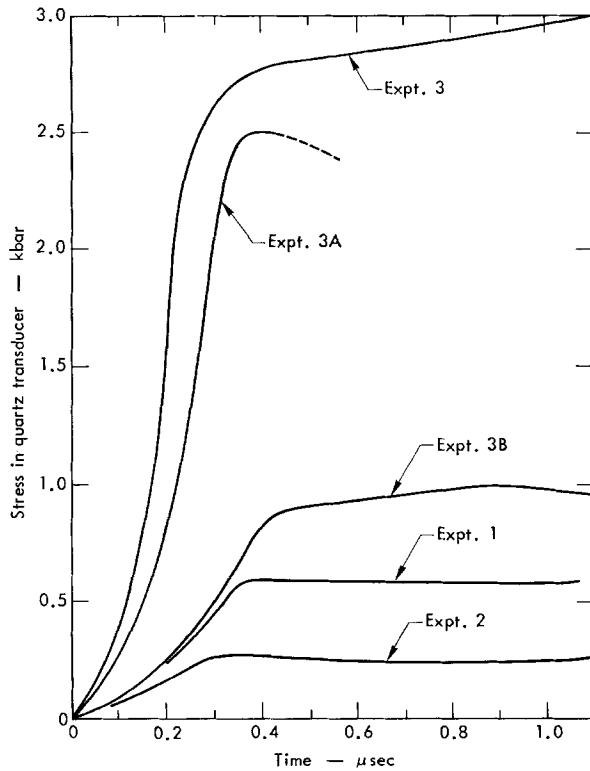


Fig. 4. Output stresses in various foams. Data are normalized to a plate velocity of 0.3 mm/ $\mu$ sec. The dashed line indicates a questionable portion of the record.

thickness, and its density (note that velocity is not a parameter here). One can observe from Figures 4 and 5 that, for a given foam, the thicker the specimen, the greater the attenuation. Also, in general, the less the bulk density, the greater the stress reduction from impedance mismatch and the greater the attenuation. There is an exception in the last observation when the results from the polystyrene bead foam are considered, although in both experiments 5 and 6, the transducer traces obtained produced somewhat questionable interpretations. These uncertainties arose because the oscilloscopes triggered in the internal mode only and the voltage sensitivities required were extreme (see below), thus making it difficult to determine the proper beginning portions of the records. Further work is intended with these materials.

On a plot of peak shock pulse stress generated in a given foam versus specimen thickness, the effect of attenuation of the pulse can be readily observed. Moreover, the extrapolation of such a plot to zero specimen thickness represents a point on the given foam's Hugoniot curve. Figure 6 presents the results from the two sets of experimental data in which three points per set were available so that there could be some credence placed

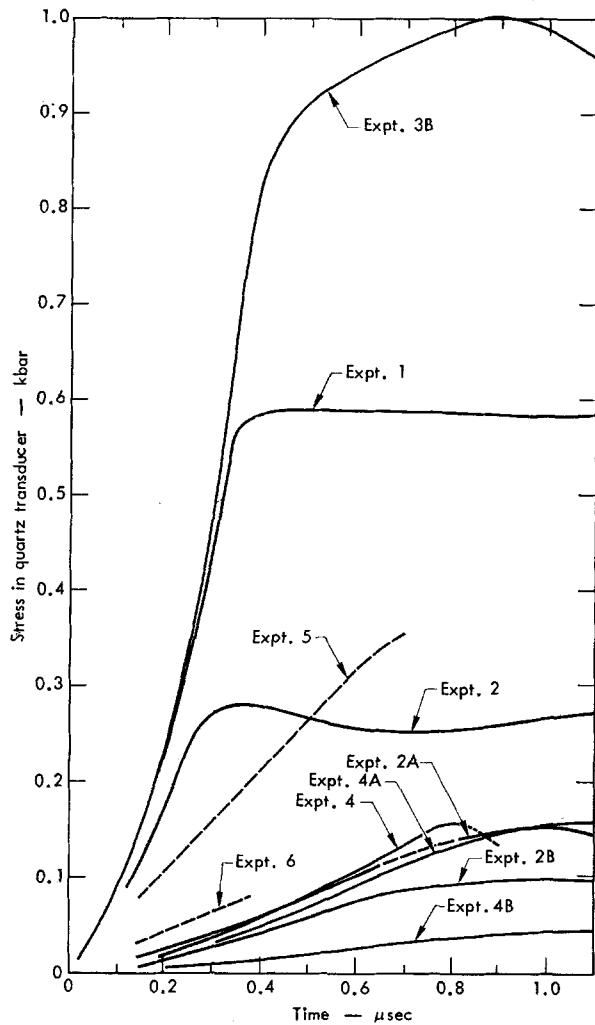


Fig. 5. Output stresses in various foams. A dashed line indicates a record, or portion thereof, that is questionable. Data are normalized to a plate velocity of 0.3 mm/ $\mu$ sec.

upon any extrapolation made. The stresses are given in reference to foam using the approximate technique stated above, namely, by dividing the ordinates in half. It is seen that the extrapolations for the two sets of data are approximately consistent with the other results shown.<sup>12</sup>

It is apparent from Table II that significant variations exist among some of the values for shock and acoustic wave velocity. Because of the limited number of tests conducted, the low stress levels in the foam, the highly attenuating nature of the materials, and the short length of travel over which such velocities were measured (in both the field experiments and the ultrasonic tests), the results with respect to velocity should be considered

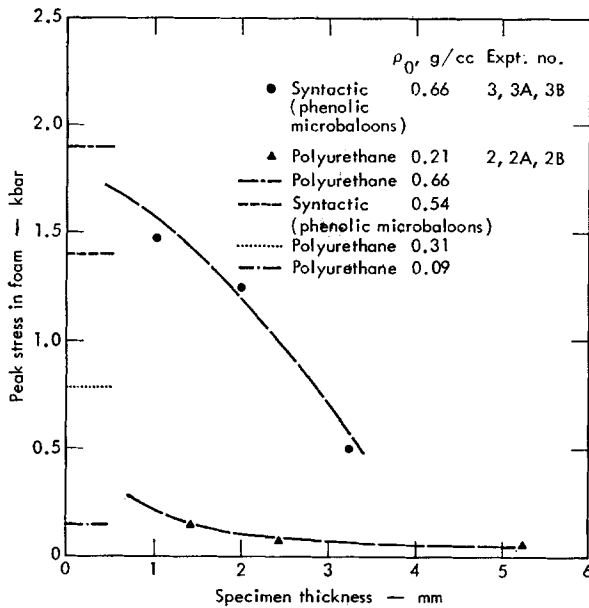


Fig. 6. Peak pulse stresses vs. specimen thickness. The approximate Hugoniot data for the foams are obtained from ref. 12. The data are normalized to a plate velocity of 0.3 mm/ $\mu$ sec.

approximate, and it is not appropriate to comment more than briefly upon the existence of trends based upon the values given. One might expect, because of the apparently low wave velocity in the coated syntactic foam, assuming that the sound velocities in the various compressed materials of similar density are about the same, that this material of those investigated would produce the greatest attenuation. (See the qualitative discussion of the attenuation process in the Introduction.) It is seen from Figure 5 that such is indeed the case.

One can also observe from Table II that the experimental values for the shock velocity increase with increasing specimen thickness for a given foam (with one minor exception). The reason for this phenomenon is not completely understood. However, a possible explanation can be found from the quasi-static deformation behavior of foams. Observations show that specimens of foam materials deform more easily in regions close to their loaded surfaces.<sup>15</sup> This behavior is not unexpected if one assumes that deformation results primarily from buckling of the cell walls. Cell walls terminating at a surface boundary have less end support than those in the interior and hence are more susceptible to buckling. Because of such a situation, the modulus or stiffness increases away from the surfaces being loaded. Since wave velocity is directly proportional to the square root of the modulus, an increase in average velocity would be expected with increase in specimen length.

In connection with making the above several observations, it is noted that the stress levels of the output pulses as shown in Figure 5 are at the very low end of the response spectrum with respect to the usual proper functioning of quartz transducers,<sup>6,7</sup> except possibly for the results obtained in experiments 3 and 3A. The actual traces at these low levels exhibited some noise because of the extreme voltage sensitivities required—in some cases as low as 0.005 V/cm and usually as low as 0.010 to 0.020 V/cm. However, with care, the traces were reduced with reasonable confidence. As far as the authors are aware, use of quartz operating as a current generating transducer has not been made at stresses as low as those reported herein. We are examining other experimental methods to monitor such low voltages (and hence stresses) as a function of time in foam materials, e.g., the use of quartz gauges using line drivers with specially designed circuitry<sup>16</sup> and quartz transducers operating in the charge mode.<sup>17</sup>

Although it would have been interesting to have posttest examinations of the targets, the present experiments were not designed to recover the specimens after impact. "Soft" recovery is difficult with these types of targets since the body of the projectile and/or the gases that follow can cause significant additional damage to a sample of relatively low strength.<sup>5</sup> This follow-on damage in most situations is a problem to separate from the effects of the original shock wave transit through the specimen.

## CONCLUSIONS

From the data presented, significant stress mitigation and pulse elongation were shown to occur when a one-dimensional strain pulse of varying amplitude and an approximate duration of 0.1  $\mu$ sec was applied, in separate experiments, to a variety of foam specimens. Moreover, this mitigation was very severe for even the thinnest sample examined (nominally 1.0 mm). It was found that the peak stress in the several experiments was reduced by factors of between about 8 and 500, depending upon foam parameters of type, density, and sample thickness.

The total stress reduction involved effects from three primary mechanisms, the flyer plate-specimen impedance mismatch, the decay of the stress pulse because of energy dissipation, and the attenuation arising from rarefactions overtaking and relieving the shock front. The approximate division of the first effect and the second and third effects are most easily seen from Figures 2 and 6. The pulse duration increases in the experiments that were considered valid were all greater than the writing time of the quartz transducer, namely, greater than 1.1  $\mu$ sec.

Although the work is at present considered somewhat preliminary, it is evident that the use of foam under conditions similar to those reported can be expected to be particularly effective in mitigating the effects of damage to structures and devices arising from shock pulse loading. Because of the physical nature of these foamed materials, the low stresses generated, and experimental difficulties (particularly with the very thin specimens), fur-

ther work is required before statements of statistical confidence of the results can be given.

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